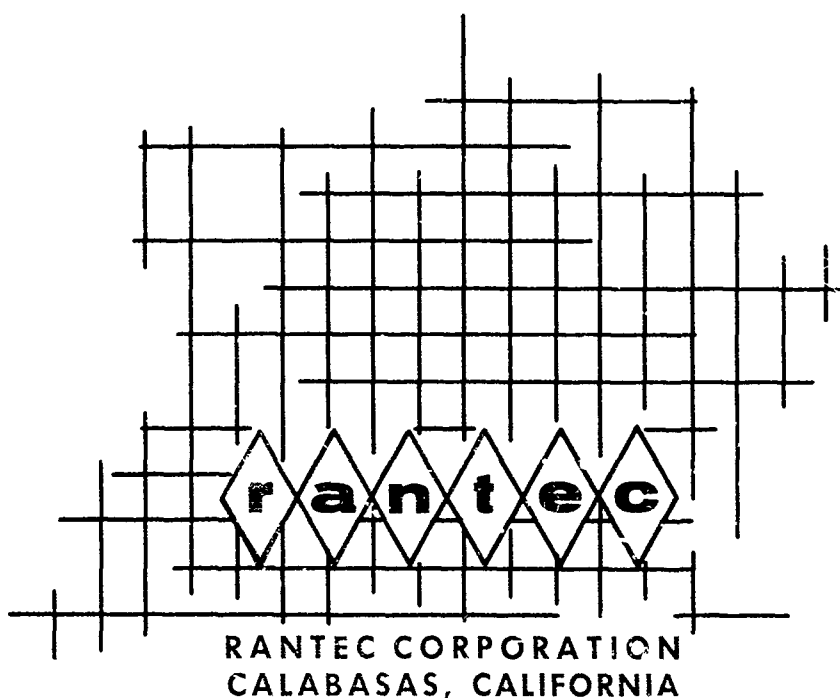Report No. 6
Sixth Quarterly Report

Covering the Period
1 December 1964 to 28 February 1965

Investigation of
MICROWAVE DIELECTRIC-RESONATOR FILTERS

Prepared for:
U. S. ARMY ELECTRONICS LABORATORIES
FORT MONMOUTH, NEW JERSEY

CONTRACT NO. DA-36-039-AMC-02267(E)
TASK NO. IP6 22001 A 057 02

By: S. B. Cohn and E. N. Torgow

rantec

RANTEC CORPORATION
CALABASAS, CALIFORNIA

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Rantec Project No. 31625

Approved:

SEYMOUR B. COHN, Technical Director

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LIST OF ILLUSTRATIONS

SECTION I

PURPOSE

This program is intended to study the feasibility of high-dielectric-constant materials as resonators in microwave filters, and to obtain design information for such filters. Resonator materials shall be selected that have loss tangents capable of yielding unloaded Q values comparable to that of waveguide cavities. The materials shall have dielectric constants of at least 75 in order that substantial size reductions can be achieved compared to the dimensions of waveguide filters having the same electrical performance.

SECTION II

ABSTRACT

The effect of metal-wall proximity on resonant frequency and
unloaded Q of a dielectric resonator is considered. It is shown that
nearby walls, such as those of a surrounding nonpropagating wave-
guide, raise the resonant frequency and degrade the unloaded Q. A
rigorous solution to the problem is not feasible, and therefore an ap-
proximate analytical approach based on the proximity of a single in-
finite plane metal wall is proposed instead. The effect of the wall on
resonant frequency is computed first. Then a perturbation method is
applied to obtain a formula for the Q of the resonator loaded by the
dissipation loss on the metal wall. Next the shift in resonant frequency
and the unloaded Q of a resonator in a rectangular waveguide are ob-
tained by superimposing the effects of the individual four walls. A
comparison of calculated and measured data shows good agreement
in the transverse-orientation case. However, agreement in the axial-
orientation case is poor. It is indicated that certain sources of error
tend to cancel in the transverse case, while adding in the axial case.
Further discussion of the axial case will be given in the next quarterly
report.

The results of further investigation of band-stop filters in TEM
line are contained in this report. In the Fifth Quarterly Report on this
program, measured response curves were given for an experimental
filter containing three dielectric resonators spaced at three-quarter-
wavelength intervals along a slab line. The filter has been reassem-
bled utilizing quarter-wavelength spacings. The resulting response
curves show no direct-coupling effects between the resonators. There-
fore, quarter-wavelength rather than three-quarter-wavelength spac-
ings are practical in the particular filter geometry that was investigated.

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A general criterion is given that can serve as a guide for determining the minimum allowable spacings of resonators in band-stop filters.

The dielectric constant and unloaded Q was measured for several lots of TiO_2 ceramic samples. The results are tabulated, and are found to be consistent with data taken on previous lots.

SECTION III

CONFERENCES

On 5 February 1965, a conference was held at U. S. Army Electronics Laboratories, Ft. Monmouth, New Jersey. Those attending were Dr. S. B. Cohn and Mr. E. N. Torgow of Rantec Corporation and Messrs. J. Agrios, N. Lipetz, and E. A. Mariani of the U. S. Army Electronics Laboratories. Progress during the fifth quarter and plans for the sixth quarter were reviewed.

On 24 March 1965, the above personnel met at the IEEE International Convention in New York, New York and held an informal discussion of progress on this program.

SECTION IV

FACTUAL DATA

1.   Introduction

In the Third Quarterly Report[1] on this program, experimental
data were given for the unloaded Q and resonant frequency of a trans-
verse-oriented resonant dielectric disk as a function of the cross-
section dimensions of a surrounding square waveguide. An approximate
analysis of these proximity effects is carried out in the present report.

A band-stop filter configuration utilizing dielectric resonators
spaced along a TEM line was investigated in the Fifth Quarterly Report.[2]
The filter described in that report had three-quarter-wave center-to-
center spacings of the resonators. During the past quarter, quarter-
wave spacing was investigated and shown to be practical with the par-
ticular parameters used in the experiment. In other band-stop filters,
three- or even five-quarter-wavelength spacings may be necessary to
avoid deterioration of the response characteristic in the rejection band.
Therefore, a rough criterion for minimum allowable spacing is given
in this report for use in general band-stop-filter design problems.

Dielectric-constant and unloaded-Q data were taken on samples
of TiO_2 ceramics made by U. S. Army Electronics Laboratories from
raw materials supplied by two different manufacturers. The results
indicate that significant differences exist between the two groups of
samples.

2.   Effect of Metal-Wall Proximity on f_o and Q_u

   a.   Basis of the Approach to the Problem

          A dielectric resonator has external field components that
decay very rapidly with distance from the resonator's surface. If a
metal wall is placed near the resonator, an electric current distribu-
tion will occur on the metallic surface in accordance with the tangential
component of magnetic field. Because of the resistivity of the metallic
surface, power will be dissipated, resulting in a degradation of the
resonator's Q factor. A second effect of the metal-wall proximity is
a change in resonant frequency. An experimental investigation of these
two effects was described in the Third Quarterly Report.[1] An approx-
imate analysis will now be given.

(a) Transverse   (b) Axial
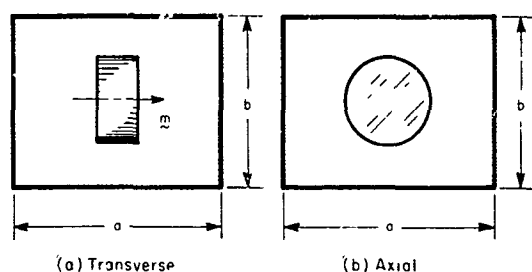
Figure 2-1. Dielectric-Disk Res-
onators in Two Basic Orientations
with Respect to a Rectangular
Waveguide.

          Figure 2-1 shows two con-
figurations of practical interest
for band-pass filters. In (a) the
dielectric disk is oriented with
its axis transverse to a non-
propagating metal waveguide,
while in (b) the orientation is
axial. A rigorous computation
of metal-wall losses and the ef-
fect on resonant frequency would
be extremely difficult and lengthy
for either of these cases. An analytical approach based on a simpler
geometry was therefore chosen, and the following approximations were
made. (1) The external field of a resonant dielectric disk was assumed
to be that of an elementary magnetic dipole located at its center and
directed parallel to its axis. (This approximation was discussed and
justified in the First Quarterly Report.[3]) (2) A single infinite plane

-6-

(a) Parallel   (b) Perpendicular
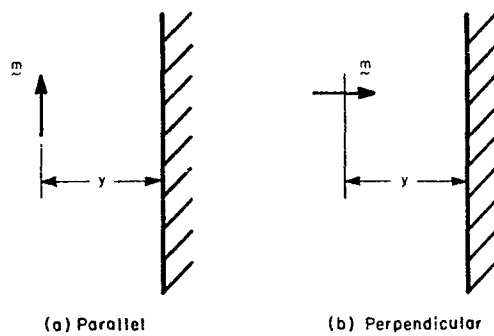
Figure 2-2. Basic Orientations of
a Magnetic Dipole with Respect to
a Plane Metal Wall

metal wall was considered. (3) The
effects of the four plane walls in
Figure 2-1 were assumed to be a
linear superposition of the effects
of the four individual walls.

b.   Analysis for Infinite
Metal Wall

Two basic infinite-
wall cases are shown in Figure 2-2.
The equivalent dipole moment $\underset{\sim}{m}$ of
the dielectric resonator is parallel to the wall in (a), and is perpen-
dicular in (b).   The power dissipated on the metal wall, P_w, is given
by

$$P_w = \frac{1}{2} R_s \iint |H_t|^2 \, dS \qquad (2-1)$$

where R_s is the surface resistivity in ohms per square on the metal
wall, H_t is the tangential magnetic field on the wall, and the integra-
tion is carried out over the infinite surface of the wall. (P_w is average
power and H_t is peak field during an R-F cycle.)   The resonator Q re-
sulting from the wall loss P_w will be designated Q_w, and is as follows

$$Q_w = \frac{\omega_o W_H}{P_w} \qquad (2-2)$$

where $\omega_o = 2\pi f_o$ and W_H is the peak value of total stored magnetic
energy during a cycle.

The difficult integration in Eq. 2-1 can be avoided by
the following procedure. Consider the equal pair of parallel magnetic

-7-
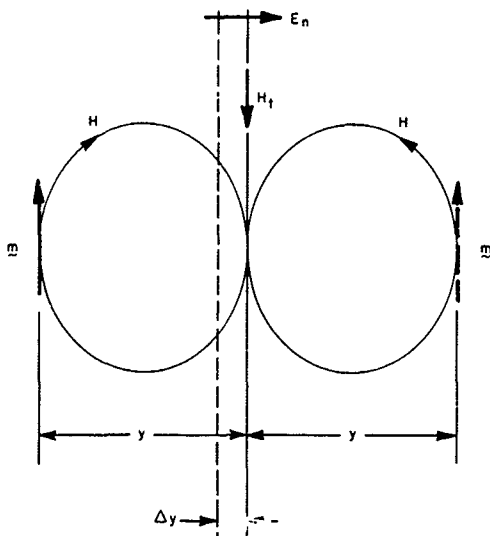
Figure 2-3. Parallel Pair of Magnetic Dipoles

dipoles in Figure 2-3. In the plane midway between them, the total H and E fields obey these conditions:

$$\underset{\sim}{H} \cdot \underset{\sim}{n} = 0 \qquad (2\text{-}3)$$

$$\underset{\sim}{E} \times \underset{\sim}{n} = 0 \qquad (2\text{-}4)$$

where $\underset{\sim}{n}$ is a unit vector normal to the plane. Hence $\underset{\sim}{H}$ is tangential and $\underset{\sim}{E}$ is perpendicular to the plane. Thus the boundary conditions of a metal wall are satisfied, and a metal wall may be placed in the mid-plane without affecting the fields. The same conclusion is obtained if the dipole on the right is considered to be the image in the wall of the dipole on the left. The essential point is that the problem of a dielectric resonator near a metal wall may be replaced by the problem of a pair of dielectric resonators.

The two dipoles in Figure 2-3 are mutually coupled by their magnetic fields. There are two basic modes of resonance of the pair of dipoles. These are illustrated by the equivalent-circuit cases of Figure 2-4 in which the resonant magnetic dipoles are replaced by loops resonated by capacitors. The self inductance of each loop is L and the mutual inductance between the loops is L_m. In Figure 2-4a, the loops are assumed to be excited by equal currents circulating in opposite directions. For this mode of excitation, the resonant frequency is

$$f_1 = \frac{1}{2\pi\sqrt{(L + L_m)C}} = \frac{f_o}{\sqrt{1 + k}} \qquad (2\text{-}5)$$

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In Figure 2-4b, the currents are equal and co-directed, and the resonant frequency is

$$f_2 = \frac{1}{2\pi\sqrt{(L - L_m)C}} = \frac{f_o}{\sqrt{1 - k}} \qquad (2-6)$$
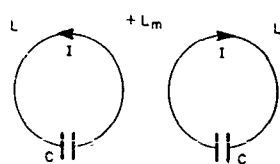
$$f_1 = \frac{1}{2\pi\sqrt{(L+L_m)C}}$$

(a) Opposite-directed Currents

$$f_2 = \frac{1}{2\pi\sqrt{(L-L_m)C}}$$

(b) Co-directed Currents

Figure 2-4. Basic Modes of Resonance of a Pair of Resonant Loops

Note that f_o is the resonant frequency of an isolated loop and k is the coupling coefficient:

$$f_o = \frac{1}{2\pi\sqrt{LC}} \qquad (2-7)$$

$$k = \frac{L_m}{L} \qquad (2-8)$$

For k small (≤ 0.1),

$$f_1 = f_o\left(1 - \frac{k}{2}\right) \qquad (2-9)$$

$$f_2 = f_o\left(1 + \frac{k}{2}\right) \qquad (2-10)$$

and, as should be expected,

$$k = \frac{f_2 - f_1}{f_o} \qquad (2-11)$$

A loop conductor is equivalent to a magnetic dipole as follows:[4]

$$m = IA \qquad (2-12)$$

-9-

where I is the current and A the area of the loop. The direction of $\underset{\sim}{m}$ is perpendicular to A according to the right-hand rule with respect to the direction of circulation of I. Thus, the pair of loops in Figure 2-4b is equivalent to a pair of parallel dipoles having the same orientation, while in Figure 2-4a the dipoles are parallel but with opposite orientation. Figure 2-3 therefore corresponds to the case of Figure 2-4b. From Eq. 2-10 the change in resonant frequency when the metal wall is moved from infinity to a distance y is as follows

$$f_2 - f_o = \frac{k}{2} f_o \qquad (2-13)$$

where k is the coupling coefficient between two resonant magnetic dipoles (or resonant dielectric disks) having center-to-center spacing s = 2y, f_o is the resonant frequency in the absence of the wall and f_2 is the resonant frequency in the presence of the wall. Note that the resonant frequency is increased by the metal wall.

A relationship between P_w, the power dissipated on the wall, and k will now be obtained. By a theorem on perturbation, if the conducting boundary of a resonant cavity is moved a small distance into the cavity so as to decrease the cavity volume by ΔV, a small change in the resonant frequency will occur as follows:[5]

$$\frac{\Delta f}{f_o} = \frac{\frac{1}{2} \mu_o \int_{\Delta V} |H|^2 \, dv - \frac{1}{2} \epsilon_o \int_{\Delta V} |E|^2 \, dv}{2W_H}$$

$$= \frac{\Delta W_H - \Delta W_E}{2W_H} \qquad (2-14)$$

where H and E are the field values in the region ΔV before the boundary is moved, and W_H is the maximum magnetic stored energy during the

cycle. The quantities ΔW_H and ΔW_E are the increments of magnetic and electric stored energy existing in the volume ΔV prior to the movement of the boundary.

Consideration of the nature of the field components of the magnetic dipoles in the region between the planes y and $y-\Delta y$ in Figure 2-3 indicates that ΔW_E is very much smaller than ΔW_H, and hence to a good approximation

$$\frac{\Delta f}{f_o} = \frac{\Delta W_H}{2 W_H} \qquad (2-15)$$

Let ΔV be the volume between planes y and $y-\Delta y$. Then

$$\Delta W_H = \frac{1}{2}\mu_o \int_{\Delta V} |H|^2 \, dv = \frac{1}{2}\mu_o \left[\int_S |H_t|^2 \, dS \right] \Delta y \qquad (2-16)$$

where the symbol $\int_S$ denotes integration over the infinite plane of symmetry between the dipoles. A comparison of Eqs. 2-1 and 2-16 shows that

$$\Delta W_H = \left(\frac{\mu_o}{R_s}\right) P_w \Delta y \qquad (2-17)$$

Therefore,

$$\frac{\Delta f}{f_o} = \left(\frac{\mu_o}{2R_s}\right) \frac{P_w \Delta y}{W_H} \qquad (2-18)$$

Now consider the effect of the shift of the metal wall on Eq. 2-13. The shift Δy produces a change Δf in f_2, and a change Δk in k, as follows

-11-

$$\Delta f = \frac{\Delta k}{2} f_o = \frac{1}{2} \left| \frac{dk}{dy} \right| f_o \Delta y = \left| \frac{dk}{ds} \right| f_o \Delta y \qquad (2\text{-}19)$$

since $s = 2y$. Next combine Eqs. 2-18 and 2-19.

$$\left| \frac{dk}{ds} \right| = \frac{\mu_o}{2R_s} \cdot \frac{P_w}{W_H} \qquad (2\text{-}20)$$

When this is combined with Eq. 2-2, a formula for Q_w is obtained.

$$Q_w = \frac{\omega_o \mu_o}{2R_s} \cdot \frac{1}{\left| \frac{dk}{ds} \right|} \qquad (2\text{-}21)$$

Thus, the general result has been derived that Q_w is inversely proportional to dk/ds, where k is the coupling coefficient of a pair of dipoles spaced by a distance $s = 2y$, and y is the distance of the metal wall from the dipole under consideration. A review of the analytical steps shows that Eq. 2-21 applies to the perpendicular orientation in Figure 2-2b, as well as to the parallel orientation in Figure 2-2a. In fact, Eq. 2-21 applies to any orientation of the magnetic dipole with respect to an infinite conducting plane at distance y, as long as k is the coupling coefficient between the dipole and its image in the plane.

The coupling coefficient k will now be computed for the parallel and perpendicular orientations of the resonant magnetic dipole with respect to the infinite ground plane. The two corresponding cases of the dipole and its image are shown in Figure 2-5. The following equation, derived in the Second Quarterly Report[4], yields k for these cases as a function of the parameters of the dipole.

$$k = \frac{\mu_o m_1 H_2}{2W_{m1}} \qquad (2\text{-}22)$$

Figure 2-5. Basic Coupling-Coefficient Cases for a Pair of Resonant Magnetic Dipoles in Free Space
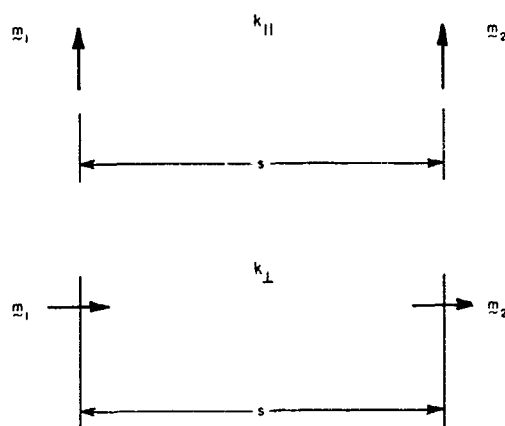
In this equation, m_1 and W_{m1} are respectively the peak magnetic dipole moment and peak magnetic stored energy of the first dipole, and H_2 is the peak magnetic field incident on the second dipole as a result of the excitation of the first dipole.

Equation 2-22 may be re-written as follows

$$k = F\frac{H_2}{m_1} \tag{2-23}$$

where

$$F = \frac{\mu_o m_1^2}{2W_{m1}} \tag{2-24}$$

The parameter F is a function only of the characteristics of the magnetic dipole. In the case of the fundamental resonance in a dielectric disk, F is related to the diameter D, length L, relative permittivity ϵ_r, and resonant wavelength λ_o by[4, 6]

$$F = \frac{0.927 D^4 L \epsilon_r}{\lambda_o^2}; \quad 0.25 \le L/D \le 0.7 \tag{2-25}$$

A more complex formula for F valid outside of the indicated range of L/D is given in the Second Quarterly Report.[4] (See Eqs. (3-48) and (3-51) of that reference.)

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The field components for a single magnetic dipole in free space are as follows.[7]

$$H_\theta = \frac{m}{4\pi r^3} \left[1 + j\beta r - \beta^2 r^2 \right] e^{-j\beta r} \sin\theta \qquad (2\text{-}26)$$

$$H_r = \frac{m}{2\pi r^3} \left[1 + j\beta r \right] e^{-j\beta r} \cos\theta \qquad (2\text{-}27)$$

$$E_\phi = \frac{30m}{r^3} \left[-j\beta r + \beta^2 r^2 \right] e^{-j\beta r} \sin\theta \qquad (2\text{-}28)$$

where $\beta = 2\pi/\lambda$ and r, θ, ϕ, are polar coordinates with respect to the dipole vector. For βr small, these formulas reduce to the static case:

$$H_\theta = \frac{m \sin\theta}{4\pi r^3} \qquad (2\text{-}29)$$

$$H_r = \frac{m \cos\theta}{2\pi r^3} \qquad (2\text{-}30)$$

$$E_\phi = 0 \qquad (2\text{-}31)$$

A numerical study of Eqs. 2-26 and 2-27 compared to Eqs. 2-29 and 2-30 indicates that the latter pair of formulas provides sufficient accuracy for the present purpose in the range $\beta r < 1.4$, or $r/\lambda < 0.2$.

In the parallel orientation case of Figure 2-2a, Eq. 2-29 gives

$$\frac{H_2}{m_1} = \frac{1}{4\pi s^3} \qquad \text{(parallel case)} \qquad (2\text{-}32)$$

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while in the perpendicular orientation case of Figure 2-2b, Eq. 2-30 gives

$$\frac{H_2}{m_1} = \frac{1}{2\pi s^3} \qquad \text{(perpendicular case)} \qquad (2\text{-}33)$$

Hence, by Eq. 2-23,

$$k_\parallel = \frac{F}{4\pi s^3} \qquad (2\text{-}34)$$

$$k_\perp = \frac{F}{2\pi s^3} \qquad (2\text{-}35)$$

and by Eq. 2-25,

$$k_\parallel = \frac{0.0739 D^4 L \epsilon_r}{\lambda_o^2 s^3} \qquad (2\text{-}36)$$

$$k_\perp = \frac{0.1478 D^4 L \epsilon_r}{\lambda_o^2 s^3} \qquad (2\text{-}37)$$

These formulas hold in the range $s \leq 0.2$ and $0.25 \leq L/D \leq 0.7$.

From Eq. 2-13, the proximity effects of the metal wall on resonant frequency in the two cases are

$$(\Delta f)_\parallel = f_2 - f_o = \frac{k_\parallel f_o}{2} \qquad (2\text{-}38)$$

and

$$(\Delta f)_\perp = \frac{k_\perp f_o}{2} \qquad (2\text{-}39)$$

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The quantity Q_w in the two cases is obtained by differentiating Eqs. 2-34 and 2-35, and substituting the results in Eq. 2-21. For both orientations

$$\frac{dk}{ds} = -\frac{3k(s)}{s} \tag{2-40}$$

Therefore, making use of $\omega_o \mu_o = 2\pi\eta/\lambda_o = 240\pi^2/\lambda_c$,

$$Q_w = \frac{40\pi^2 s}{\lambda_o R_s k(s)} \tag{2-41}$$

The surface resistivity of a smooth metallic surface is given by the following

$$R_s = 8.25(10)^{-3} r_s \sqrt{f_{Gc}} \qquad \text{ohms per square} \tag{2-42}$$

where f_{Gc} is the frequency in gigacycles per second and r_s is the surface resistivity relative to copper (i.e., $r_s = 1$ in the case of copper). Thus

$$Q_w = \frac{4050 \, s_{in} \sqrt{f_{Gc}}}{r_s k(s)} \tag{2-43}$$

where $s_{in} = 2y_{in}$ is twice the spacing in inches between the center of the magnetic dipole and the metal wall. The coupling coefficient $k(s)$ is given by Eqs. 2-36 and 2-37 for the parallel and perpendicular orientations. With this substitution the final formulas for $Q_{w\perp}$ and $Q_{w\parallel}$ are obtained.

$$Q_{w\perp} = \frac{3.825(10)^6 s_{in}^4}{r_s f_{Gc}^{1.5} D_{in}^4 L_{in} \epsilon_r} \tag{2-44}$$

-16-

$$Q_{w\parallel} = 2Q_{w\perp} \qquad (2\text{-}45)$$

These formulas are subject to the restriction $0.25 \le L/D \le 0.7$, as in Eq. 2-25.

      c.   Case of Rectangular Waveguide and Transverse Orientation

      The losses on the four walls of a rectangular waveguide surrounding a dielectric resonator will now be assumed equal to the sum of the losses on four single infinite walls, each at its respective spacing, y. Consider first the transverse-orientation case in Figure 2-1a. The two walls with perpendicular orientation have $s = 2y = a$, while the two walls with parallel orientation have $s = 2y = b$. Hence, utilizing Eqs. 2-44 and 2-45, Q_{wg} of a rectangular waveguide is

$$\frac{1}{Q_{wg}} = \frac{2}{Q_{w\perp}} + \frac{2}{Q_{w\parallel}} \qquad (2\text{-}46)$$

Now assume the useful case of a square waveguide, $a = b$. Then

$$Q_{wg} = \frac{Q_{w\perp}}{3} = \frac{1.275(10)^6 a_{in}^4}{r_s f_{Gc}^{1.5} D_{in}^4 L_{in} \epsilon_r} \qquad (2\text{-}47)$$

The complete unloaded Q, Q_u, of the resonator inside the waveguide is related to Q_{wg} by

$$Q_u = \frac{Q_{ui} Q_{wg}}{Q_{ui} + Q_{wg}} \qquad (2\text{-}48)$$

where Q_{ui} is the Q of the resonator loaded only by its internal losses.

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In the Third Quarterly Report[1], experimental Q_u data was given for a dielectric resonator in a series of square waveguides of various dimensions and wall materials. The resonator had parameters D = 0.430", L = 0.220", and ϵ_r = 87.1. Its value of Q_{ui} had been determined as 11,650 by the rejection technique in WR-284 waveguide. (Wall losses in this large waveguide would be expected to be negligible.) The measured Q_u and f_o points are plotted versus the cross-section dimension, a, of the square waveguide in Figures 2-6 and 2-7.
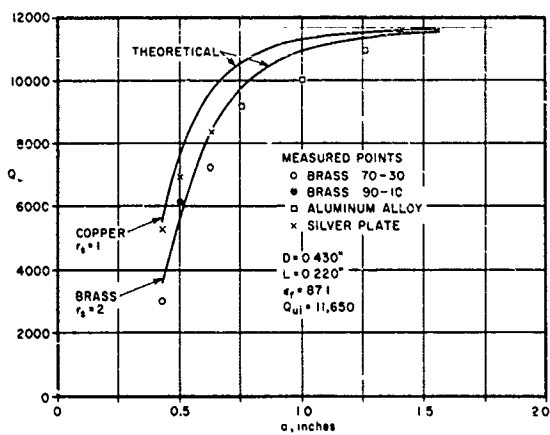
Figure 2-6. Proximity Effect on Q_u of a Dielectric Resonator in a Square Waveguide - Transverse Orientation
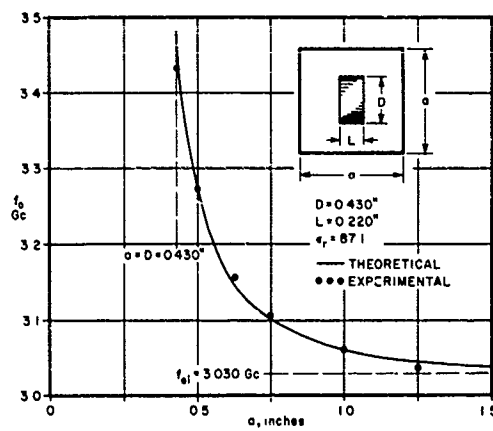
Figure 2-7. Proximity Effect of Square Waveguide on Resonant Frequency of a Dielectric Disk - Transverse Orientation

Curves of Q_u-versus-a were computed by means of Eqs. 2-47 and 2-48, and are plotted in Figure 2-6 for two values of r_s: $r_s \approx 1$ applying to copper and silver, and $r_s \approx 2$ applying to yellow (70-30) brass. A curve for aluminum alloy ($r_s \approx 1.5$) is not plotted, but would fall approximately midway between the two curves in the figure. The values of f_{Gc} used in the computation were the actual measured resonant frequencies plotted versus the a dimension in Figure 2-7.

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Agreement between the theoretical curves and experimental values of Q_u is seen to be quite good. The discrepancy for large values of a may be due to an error in the measurement of Q_{ui}. A value of 10,500 to 11,000 instead of 11,650 would result in a major improvement. However, the disagreement may simply be the result of the various approximations and assumptions in the analysis.

A theoretical formula for small changes in f_o due to the walls of a rectangular waveguide can be obtained from Eqs. 2-38 and 2-39 by assuming validity of superposition as in the case of Q_{wg}. Thus,

$$f_{o_{wg}} - f_{o_i} = (\Delta f)_{wg} = 2(\Delta f)_{\parallel} + 2(\Delta f)_{\perp} = (k_{\parallel} + k_{\perp})f_{o_i} \qquad (2\text{-}48)$$

where f_{o_i} and $f_{o_{wg}}$ are the resonant frequencies of the isolated and enclosed resonators, respectively, and $k_{\parallel}$ and $k_{\perp}$ are given by Eqs. 2-36 and 2-37. If the change in f_o is not small, better results are obtained by using the exact formula Eq. 2-6 as the basis of the derivation rather than the approximate formula Eq. 2-10. In Eq. 2-6, the coupling coefficient values may be superimposed for the four walls as follows

$$f_{o_{wg}} = \frac{f_{o_i}}{\sqrt{1 - 2k_{\perp} - 2k_{\parallel}}} \approx \frac{f_{o_i}}{1 - k_{\perp} - k_{\parallel}} \qquad (2\text{-}49)$$

In the case of square waveguide, $a = b = s$ and $k_{\perp} = 2k_{\parallel}$. Hence

$$f_{o_{wg}} \approx \frac{f_{o_i}}{1 - 3k_{\parallel}} \qquad (2\text{-}50)$$

where $k_{\parallel}$ is evaluated from Eq. 2-36 using $s = a$ and $\lambda_o = \lambda_{o_i}$.

-19-

Equations 2-50 and 2-36 have been applied to the experimental example described above, and the resulting curve is shown in Figure 2-7 compared to the measured points. Agreement is seen to be good. Equation 2-48 yields a value about 10% lower than Eq. 2-50 at the smallest value of a, while the square-root-denominator version of Eq. 2-49 is about 10% high. The use of Eq. 2-50 rather than the other formulas is therefore recommended.

d.   Case of Axial Orientation

In the case of a dielectric resonator having axial orientation in a rectangular waveguide, the four walls are parallel to the equivalent magnetic-dipole vector. Hence, superposition of losses leads to the following for square waveguide,

$$Q_{wg} = \frac{Q_{w\|}}{4} = \frac{Q_{w\perp}}{2} \qquad (2-51)$$

Comparison with Eq. 2-47 shows that Eq. 2-51 may be written as below

$$Q_{wg} = \frac{1.912(10)^6 a_{in}^4}{r_s f_{Gc}^{1.5} D_{in}^4 L_{in} \epsilon_r} \qquad (2-52)$$

Similarly, the effect of the waveguide walls on resonant frequency may be computed from

$$f_{o_{wg}} = \frac{f_{o_i}}{1 - 2k} \qquad (2-53)$$

where $k_\|$ is evaluated from Eq. 2-36 with $s = a$ and $\lambda_o = \lambda_{o_i}$.

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On the basis of limited experimental data, Eq. 2-52 appears to be in error on the high side. The few values of Q_u measured in the axial orientation appear to correlate better with Eq. 2-47, rather than with Eq. 2-52. When the various approximations in the analysis are examined carefully, the better accuracy achieved with the transverse orientation can be explained in terms of certain sources of error cancelling, while with axial orientation these sources of error add. The same conclusion applies to Eq. 2-53 for $f_{o_{wg}}$, for which a large error was also observed. Further discussion of Q_{wg} and $f_{o_{wg}}$ in the axial orientation will be postponed until the next report, when more complete experimental data will be available.

3.   Band-Stop Filters in TEM Lines

    a.   Experimental Results with Quarter-Wavelength Spacings

        A three-resonator band-stop filter in slab line was described in the last quarterly report. The adjacent resonators in this filter had been located on opposite sides of the center conductor and spaced at 3/4 wavelength intervals along the line to minimize direct coupling between resonators. As a further precaution, the filters had contained shorting posts between the ground planes to shield the resonators from each other. It was observed during the measurement of the filter characteristics that performance was not affected when the shorting posts were removed.

        Since coupling between adjacent resonators was not significant in the use of 3/4 wavelength resonator spacings in the experimental band-stop filter, a reduction in the resonator spacing to 1/4 wavelength appeared to be practical. Such a reduction would appreciably reduce the over-all length of the filter. A three-resonator filter was therefore assembled, with alternate resonators placed on

opposite sides of the center conductor. Shorting posts were again provided between ground planes in the vicinity of the resonators. However, the resonator spacing for this filter was a quarter wavelength at the center frequency. The design 0.01-db-ripple bandwidth was 32.5 Mc (3-db bandwidth = 17.3 Mc) at a center frequency of 3400 Mc. The ground-plane spacing was 0.430 inch, as in the earlier experiments. The center-to-center distance between each resonator and the center conductor of the slab line was initially set to provide a value of unloaded Q, Q_L, approximately equal to the theoretical value of Q_{ex} required for that resonator. The height of the center resonator was sanded down to adjust its resonance very close to the desired center frequency with the tuning screw removed. The end resonators, being slightly higher, resonated at a frequency just below the specified stop band, and could easily be adjusted by means of the tuning screws. The effect of the tuning screw upon resonator Q was neglected in the initial placement of the resonator. Minor adjustments in the positions of the tuning screws and the resonators were necessary during the alignment of the filter.

The insertion-loss characteristics of the quarter-wave-coupled, three-resonator filter is shown in Figure 3-1. When the shorting posts around the resonators were removed, the filter characteristics shifted by less than 1 Mc. The distortion of the response characteristic was insignificant. Therefore, it was concluded that the coupling between adjacent resonators is sufficiently weak so that it can be neglected for all practical rejection bandwidths.

The unloaded Q, Q_u, of a single resonator coupled to the slab line was determined by measuring the 3-db return-loss bandwidth (i.e., bandwidth for reflection coefficient ≥ 0.707), the center frequency, and the peak insertion loss of the center resonator. All tuning screws were removed during this measurement. The loaded Q, Q_L,
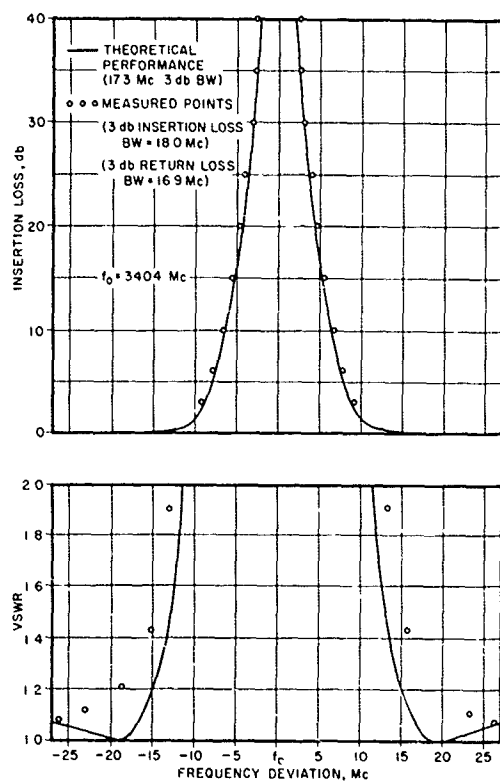
Figure 3-1. Response of Three Resonator Band-Stop Filter in Slab Line - λ/4 Spacing Between Resonators

was determined, from the band-width and center-frequency measurements, to be $Q_L = 201$. The peak insertion loss was 27 db. Therefore, using the relationship,[3]

$$L_{max} = 20 \log\left(\frac{Q_u}{Q_L}\right) \quad db,$$

Q_u was found to equal 4500. This agreed closely with the value of $Q_u = 5000$ assumed in the Fifth Quarterly Report.

It can be shown that the absolute 3-db return-loss bandwidth of a rejection resonator (with a high, but finite, unloaded Q) is slightly narrower than the bandwidth determined from the external Q. On the other hand, the 3-db insertion-loss bandwidth is almost exactly that determined by the loaded Q. In the case of the three-resonator filter described above, it was found that the 3-db bandwidths were:

<div style="text-align:center">

3-db design bandwidth = 17.3 Mc

Measured 3-db return-loss bandwidth = 16.9 Mc

Measured 3-db insertion-loss bandwidth = 18 Mc

</div>

The loaded Q's of the resonators in the final filter configuration were determined from their positions with respect to the center conductor of the slab line (see Figure 3-2). For the center resonator, Q_L was 201. For the end resonators, Q_L was 340. These values of Q_L were

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Figure 3-2. Measured Q_L - Dielectric Resonator in Slab Line

determined neglecting the effects of tuning screws. As described above, the center resonator required very little tuning, so that the influence of the tuning screw upon Q_L could be neglected. The tuning screws for the end resonators, however, were close enough to the resonators to have a pronounced effect. From data presented in the last report, it was shown that a tuning screw in close proximity to a resonator, set for a loaded Q close to that of an end resonator, would tend to reduce that Q_L by approximately 4%. Therefore, it is reasonable to assume that the loaded Q's of the end resonator in the final filter were close to 325.

The following table compares the values of Q_L and Q_{ex} (determined from Q_L and the measured value of Q_u) to the theoretical values for several design bandwidths. Good agreement exists between the measured values of Q_L and the theoretical (lossless case) values of Q_{ex} for the observed 3-db insertion-loss case. Similar agreement exists between the measured values of Q_{ex} and the theoretical values of Q_{ex} for the observed 3-db return-loss bandwidth.

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TABLE 3-1

	Q_L Measured	Q_{ex} Measured	Q_{ex} (Theoretical) 3-db Bandwidth*		
			16.9 Mc	17.3 Mc	18.0 Mc
End Resonators	325	350	340	331	320
Center Resonator	201	210	220	214	207

$f_o = 3400$ Mc

*Ripple bandwidth – 1.882 x (3-db bandwidth)

b. General Criterion for Minimum Resonator Spacing

In the preceding paragraphs it was demonstrated that
direct coupling between adjacent resonators was insignificant for the
case considered. This direct coupling will now be examined in order
to establish a criterion for minimum center-to-center spacing between
adjacent resonators. While this criterion is specifically directed to
the case of a dielectric resonator whose axis is normal to a pair of
parallel ground planes, the principles considered are generally ap-
plicable to any type of rejection resonator spaced along any type of
transmission line or waveguide.

Optimum coupling between TEM lines and dielectric
resonators occurs when the axis of the resonator is transverse to the
axis of the TEM line and parallel to the H field of the line. In the case
of a slab- or strip-line structure, the resonator should generally be
centered between the ground planes with its axis normal to the planes.
In this case, the leading contributor to direct coupling between reso-
nators is the nonpropagating mode whose E field is parallel to the
ground planes and exhibits a half-wave variation in amplitude from
one plane to the other. This parallel-plate mode is therefore analogous

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to the cut-off TE_{10} mode in a nonpropagating square waveguide, insofar as coupling between resonators located within the guide is concerned.

The coefficient of coupling, k, between two resonators relates the voltage induced in the second resonator to the excitation current in the first. Hence, the direct coupling between two resonators in a band-stop filter is on the order of $20 \log 1/k$ db. In the Second and Third Quarterly Reports[4,1] it was shown that the leading term of the coupling-coefficient formula for two resonators in the transverse orientation in a square waveguide decreases in proportion to the factor $e^{-\alpha s}$, where α is the attenuation constant of the cut-off TE_{10} mode. An examination of the various computed coupling-coefficient curves in these reports shows that the leading term approaches a value of about 0.1 when the center-to-center spacing between resonators goes to 0. Thus, the actual direct coupling between resonators is approximately 20 db below the attenuation predicted by the factor, αs. If the criterion for minimum coupling is based upon αs, then an added safety margin of approximately 20 db will be realized for the direct coupling between resonators.

As a general rule of thumb it is desirable that the attenuation, αs, be somewhat greater than the minimum rejection required of each resonator. Thus, for a filter containing n resonators and having a total length equal to $(n-1)s$,

$$(n-1)\alpha s \geq (L_s)_{min} \qquad (3-1)$$

where $(L_s)_{min}$ is the minimum rejection specified for the filter. Equation 3-1 can be applied in general as a rough criterion for determining the minimum permissible spacing in band-stop filters of any type. In using Eq. 3-1, α is the smallest attenuation constant of the various nonpropagating modes excited by the resonator.

The attenuation constant for the lowest-order nonpropagating TE mode in the parallel-plate structure is,

$$\alpha = \frac{27.3}{b} \qquad \text{db per inch,} \qquad (3-2)$$

where b is the ground plane spacing in inches. Therefore, the minimum spacing between the adjacent resonators is approximately

$$s_{min} = \frac{b\,(L_s)_{min}}{27.3(n-1)} \qquad \text{inches} \qquad (3-3)$$

$(L_s)_{min}$ should be at least equal to the highest rejection which must be maintained at any point in the stop band.

In the case of the three-resonator filter described above, it can be seen that the minimum spacing allowed by Eq. 3-3 is considerably smaller than one-quarter wavelength, even when the minimum rejection is required to be greater than 100 db. The spacing between adjacent resonators of concern here is actually the half-wavelength spacing between the first and third resonators. Resonators were placed on alternate sides of the center conductor, which serves effectively as a shield to eliminate direct coupling between the center resonator and either of the end resonators. This does not mean, however, that shorting posts between the ground planes surrounding each resonator can be eliminated. Such posts serve not only to further attenuate nonpropagating modes, but also serve to attenuate any undesired propagating mode. In the case of slab line or strip line, an extraneous propagating mode between the parallel ground planes can easily be excited by vertical asymmetries, such as a slight displacement of a dielectric resonator closer to one ground plane than the other. Therefore, it is generally good engineering practice to provide shorting posts

or screws between ground planes in those regions of a parallel-ground-plane TEM line where vertical asymmetries can occur. This is especially necessary in filters where stop-band insertion losses greater than 40 or 50 db are desired.

4.   Dielectric-Constant Measurements

Several lots of TiO_2 dielectric were received from the U. S. Army Electronics Laboratories. Several resonators were ground from each type of material that was separately identified in each lot. Data for these samples are given in Table 4-1.

The relationship between dielectric constant and sample density of the Glidden sample (K-T-20) correlates closely with the curve shown in the previous report. * All of the Baker samples exhibited greater densities for the observed dielectric constants than could be predicted by this curve. However, these densities and dielectric constants agreed closely with that of another Baker sample described in the previous report.

TABLE 4-1

DIELECTRIC MATERIAL CHARACTERISTICS

Sample	Firing Process	Sample Size	ϵ_r	Density gm/in³	Q_u†
Baker					
K-T-18-(2)	2550°F for 1 hr, 10,000 psi with binder	.400 D x .250"L / .400"D x .250"L	104.8 / 104.8	68.0 / 68.0	4950 / 3870
K-T-18-(1)	2550°F for 1 hr, 10,000 psi	.400"D x .100"L / .400"D x .100 L	103.0 / 103.0	68.6 / 68.6	4870 / 4470
K-T-17	2550°F for 1 hr	.400"D x .250"L / .400"D x .250"L	104.8 / 104.8	68.1 / 68.0	5630 / 3720
K-T-12	2600°F for 2 hrs, regular fired	.400"D x .150"L / .400"D x .120"L	103.0 / 103.3	68.6 / 68.1	4630 / 5010
K-T-16	2600°F for 1 hr, regular fired	400"D x .100"L / 400"D x .100"L	103.2 / 103.2	66.3 / 67.8	4250 / 4130
Glidden					
K-T-20	2850°F for 1 hr	.400"D x .100"L / .400"D x .100"L / .400"D x 250"L	95.0	64.1 / 64.1 / 63.2	7800 / 7960 / 7710

NOTE All samples are TiO_2
†Q_u values are the averages of several readings.

*See Figure 5-1 of Fifth Quarterly Report.

SECTION V

CONCLUSIONS

A rigorous analysis of the effect of a waveguide's walls on the unloaded Q and resonant frequency of a dielectric resonator was judged not to be feasible. An approximate approach to the theoretical problem was therefore taken. Comparison with measured data proved to be quite good in the case of the transverse orientation in a square nonpropagating waveguide, but poor in the axial case. Preliminary consideration of the sources of error in the approximations indicates these results to be reasonable. Further analysis and measurements are required, however, before the axial-orientation case can be brought to a logical conclusion.

The results on a three-resonator band-stop filter indicate that quarter-wave spacings of the resonators can be utilized rather than three-quarter-wavelength spacings without deleterious direct-coupling effects between resonators. A rough criterion is given that will permit the minimum practical resonator spacing to be estimated in the general case of a band-stop filter utilizing any type of resonator in any type of propagating transmission line or waveguide. In most situations of dielectric resonators located along a TEM-mode line, quarter-wave spacings should suffice. In a propagating waveguide, three-quarter-wavelength or even five-quarter-wavelength spacings are likely to be necessary.

Various TiO_2 samples made from powders of different manufacture prove to fall on somewhat different curves of dielectric constant versus density. This indicates that impurities have a varying effect on dielectric constant, and that the specific impurities depend upon the particular manufacturer of the raw material.

SECTION VI

PROGRAM FOR NEXT INTERVAL

Further study will be made of unloaded-Q and resonant-frequency effects of waveguide-wall proximity in the axial case. Investigation of end-loading techniques will be continued, and investigation of the band-pass configuration of an axially oriented resonator in a cut-off circular tube will be concluded. Work will start on a directional-filter structure containing a dual-mode dielectric resonator.

SECTION VII

LIST OF REFERENCES

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2.   S. B. Cohn and E. N. Torgow, "Investigation of Microwave
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        Book Co., Inc., 1961.

SECTION VIII

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Mr. Kenneth C. Kelly Senior Engineer	51
Mr. Richard V. Reed Engineer	38
Mr. Howard V. Stein Junior Engineer	322

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13 ABSTRACT

The effect of metal-wall proximity on resonant frequency and unloaded Q of a dielectric resonator is considered. It is shown that nearby walls, such as those of the surrounding nonpropagating waveguide, raise the resonant frequency and degrade the unloaded Q. Formulas are obtained for the shift in resonant frequency and the unloaded Q of the resonator loaded by the dissipation loss on the metal walls. Good agreement exists between the calculated and measured data for the case of a transversely oriented resonator. This analysis will be extended for the case of the axially oriented resonator in the next quarter.

The results of band-stop filters in TEM using quarter-wavelength spacing between resonators (instead of three-quarter wavelength) indicate that no direct coupling exists between resonators; hence, such resonator spacing is practical for the construction of band-rejection filters. Also, a general criterion is given that serves as a guide for determining the minimum allowable spacings of resonators in band-stop filters.

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Dielectric Resonator						
Band-Pass						
Band-Reject						
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